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No new expressions are obtained by subtracting these from unity or by inverting them. The study of equality among these functions becomes a special case when the group generated by the operations  $x_1 - n$ ,  $x_1^2/n$  transforms a point into less than six distinct points. This question has been completely solved for the general dihedral rotation group.

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## THE EXPRESSION OF THE AREAS OF POLYGONS IN DETERMINANT FORM.

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By R. P. BAKER.

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The area of the triangle the rectangular coördinates of whose vertices are  $(x_1, y_1)$ ;  $(x_2, y_2)$ ;  $(x_3, y_3)$ , is

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

For the area of a quadrilateral whose vertices are 1, 2, 3, 4, diagonals (13) and (24) and such that circuits (123), (134) have the area on the left, we have

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 & 0 \\ x_2 & y_2 & 1 & 1 \\ x_3 & y_3 & 1 & 0 \\ x_4 & y_4 & 1 & 1 \end{vmatrix} \equiv \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 & -k \\ x_2 & y_2 & 1 & 1-k \\ x_3 & y_3 & 1 & -k \\ x_4 & y_4 & 1 & 1-k \end{vmatrix}.$$

The case of the pentagon or polygon of more sides than 5 is different.

Suppose that the area  $P$  of the pentagon can be expressed by

$$\lambda \begin{vmatrix} x_1 & y_1 & 1 & a_1 & b_1 \\ x_2 & y_2 & 1 & a_2 & b_2 \\ x_3 & y_3 & 1 & a_3 & b_3 \\ x_4 & y_4 & 1 & a_4 & b_4 \\ x_5 & y_5 & 1 & a_5 & b_5 \end{vmatrix} = \lambda \begin{vmatrix} x_1 & y_1 & 1 & 0 & 0 \\ x_2 & y_2 & 1 & a'_2 & b'_2 \\ x_3 & y_3 & 1 & a'_3 & b'_3 \\ x_4 & y_4 & 1 & a'_4 & b'_4 \\ x_5 & y_5 & 1 & a'_5 & b'_5 \end{vmatrix}$$

the latter being obtained from the former by subtracting multiples of columns.

Expanding by Laplace's method in minors of the first three columns we get the area as a sum of multiples of triangular areas all having the point 1 as vertex. The multipliers must obviously be equal. Hence all the determinants

$\begin{vmatrix} a'_i & b'_i \\ a'_j & b'_j \end{vmatrix}$  must be equal. This is impossible, for

$$\frac{1}{2} \begin{vmatrix} a_2, & b_2, & a_2, & b_2 \\ a_2, & b_3, & a_3, & b_3 \\ a_4, & b_4, & a_4, & b_4 \\ a_5, & b_5, & a_5, & b_5 \end{vmatrix} \equiv (23)(45) - (24)(35) + (25)(34) \equiv 0,$$

which cannot be satisfied by

$$(23) = (45) = (24) = (35) = (25) = (34).$$

The general case fails in consequence of a similar identical relation among the determinants of a matrix. This relation can be expressed as the expansion of a determinant of  $2(n-3)$  rows which can be symbolized by  $\frac{A}{B} \bigg| \frac{C}{O}$ , where  $A$ ,  $B$ ,  $C$  denote, respectively,

$$\begin{vmatrix} 0, & 0, & \dots, & a_{n-2} \\ 0, & 0, & \dots, & b_{n-2} \\ \dots & \dots & \dots & \dots \\ 0, & 0, & \dots, & l_{n-2} \end{vmatrix}, \quad \begin{vmatrix} a_2, & a_3, & a_4, & \dots, & a_{n-2} \\ b_2, & b_3, & b_4, & \dots, & b_{n-2} \\ \dots & \dots & \dots & \dots & \dots \\ l_2, & l_3, & l_4, & \dots, & l_{n-2} \end{vmatrix}, \quad \begin{vmatrix} a_{n-1}, & a_n, & a_2, & a_3, & \dots, & a_{n-4} \\ b_{n-1}, & b_n, & b_2, & b_3, & \dots, & b_{n-4} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ l_{n-1}, & l_n, & l_2, & l_3, & \dots, & l_{n-4} \end{vmatrix}.$$

When this is expanded according to Laplace's method in determinants of  $(n-3)$  rows, we get

$$\begin{aligned} & (2, 3, \dots, n-3, n-2)(n-1, n, 2, \dots, n-4) \\ & - (2, 3, \dots, n-3, n-1)(n-2, n, 2, \dots, n-4) \\ & + (2, 3, \dots, n-3, n)(n-3, n, 2, \dots, n-4) = 0, \end{aligned}$$

which cannot be satisfied if these determinants are all equal.

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## DEPARTMENTS.

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### SOLUTIONS OF PROBLEMS.

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#### ALGEBRA.

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Problem 206 was also solved by J. E. Sanders; No. 207 was also solved by L. E. Newcomb.

208. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve (1)..... $x^4 + y^4 = 14x^2y^2$ ; (2)..... $x + y = m$ .

I. Solution by EDWIN L. RICH, Student at Lehigh University.

Equation (1) may be written

$$(x^2 - y^2 - xy\sqrt{12})(x^2 - y^2 + xy\sqrt{12}) = 0 \dots\dots (3).$$